

Chiral Dynamics^{1,2}

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Abstract

After a short summary of my talk, I discuss K_{l3} decays and elastic $\pi\pi$ scattering in the framework of chiral perturbation theory.

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1 Introduction

In the first part of my talk, I gave an introduction to the effective theory of QCD at low energy, called chiral perturbation theory (CHPT) [1, 2]. There are many excellent reviews and lectures on the subject available on the market - for a comprehensive list, see Ref. [3]. Therefore, I do not try to add one more here. Instead, I refer the interested reader to Refs. [3] and [4]. In the second part, I illustrated the method with a few examples. Here, I shall consider two of them, K_{l3} decays and elastic $\pi\pi$ scattering. Both processes are presently under theoretical and experimental investigation. Finally, I also presented the EURODAFNE network, outlining the work planned in that enterprise. Lack of space prevents me to cover this topic here - I refer the interested reader to the relevant homepages [5] and to *The Second DAFNE Physics Handbook* [6].

2 K_{l3} decays

The so called K_{l3} decays are

$$\begin{aligned} K^+(p) &\rightarrow \pi^0(p') l^+(p_l) \nu_l(p_\nu) & [K_{l3}^+] \\ K^0(p) &\rightarrow \pi^-(p') l^+(p_l) \nu_l(p_\nu) & [K_{l3}^0] \end{aligned} \quad (1)$$

and their charge conjugate modes. The symbol l stands for μ or e . I consider the isospin symmetry limit $m_u = m_d, \alpha_{\text{QED}} = 0$.

The matrix element for K_{l3} decays contains a leptonic and a hadronic factor. The hadronic part is

$$\begin{aligned} \langle \pi^0(p') | V_\mu^{4-i5}(0) | K^+(p) \rangle &= \langle \pi^-(p') | V_\mu^{4-i5}(0) | K^0(p) \rangle \\ &= \frac{1}{\sqrt{2}} [(p' + p)_\mu f_+(t) + (p - p')_\mu f_-(t)] . \end{aligned} \quad (2)$$

In this formula, V_μ^{4-i5} denotes the hadronic vector current, and t is the momentum transfer to the lepton pair, $t = (p' - p)^2 = (p_l + p_\nu)^2$.

The quantity f_+ is referred to as the vector form factor, because it specifies the P-wave projection of the crossed channel matrix element. The S-wave projection is described by the scalar form factor

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t) . \quad (3)$$

Analyses of K_{l3} data often assume a linear dependence

$$f_{+,0}(t) = f_+(0) \left[1 + \lambda_{+,0} \frac{t}{M_{\pi^+}^2} \right] . \quad (4)$$

2.1 Previous measurements

I refer the reader to the 1982 version of the PDG [7] for a critical discussion of the wealth of experimental information on $\lambda_{+,0}$. Here I present a short summary.

K_{e3} -experiments

The λ_+ values obtained are fairly consistent. The average values are

$$\begin{aligned} K_{e3}^+ : \lambda_+ &= 0.0286 \pm 0.0022 \quad [8] \\ K_{e3}^0 : \lambda_+ &= 0.0300 \pm 0.0016 \quad [8] . \end{aligned} \quad (5)$$

$K_{\mu3}$ -experiments

The result by Donaldson et al. [9]

$$\begin{aligned} \lambda_+ &= 0.030 \pm 0.003 \\ \lambda_0 &= 0.019 \pm 0.004 \end{aligned} \quad (6)$$

dominates the statistics in the $K_{\mu3}^0$ case. The λ_+ value (6) is consistent with the K_{e3} value (5). However, the situation concerning the slope λ_0 is rather unsatisfactory, as the following list from $K_{\mu3}^0$ decays illustrates³

$$\lambda_0 = \left\{ \begin{array}{llll} 0.019 & \pm & 0.004 & [9] \\ 0.025 & \pm & 0.019 & [10] \\ 0.047 & \pm & 0.009 & [11] \\ 0.039 & \pm & 0.010 & [12] \\ 0.050 & \pm & 0.008 & [13] \\ 0.0341 & \pm & 0.0067 & [14] . \end{array} \right. \quad (7)$$

The χ^2 fit to the $K_{\mu3}^0$ data yields $\lambda_+ = 0.034 \pm 0.005$, $\lambda_0 = 0.025 \pm 0.006$ with a $\chi^2/DF = 88/16$ [7, p.76]! The situation in the charged mode $K_{\mu3}^+$ is slightly better [7].

³The list is chronological, starting 1974, ending 1981. Earlier data may be found in Ref. [8]. More recent data are not yet available.

2.2 Theory

The theoretical prediction of K_{l3} form factors has a long history, starting in the sixties with the current algebra evaluation of $f_{\pm,0}$. For an early review of the subject and for references to work prior to CHPT evaluations of $f_{\pm,0}$, I refer the reader to [15]. Here I concentrate on the evaluation of the form factors in the framework of CHPT. The one-loop corrections have been evaluated in [16], with the result

$$\lambda_0 = 0.017 \pm 0.004, \quad (8)$$

where the error is an estimate of the uncertainties due to higher-order contributions. The prediction (8) is in agreement with the high-statistics experiment [9] quoted in (6,7), but in flat disagreement with some of the more recent data listed in (7). The double logarithms that occur at order p^6 in the K_{l3} form factors have been determined recently [17], the full two-loop calculation is under way [18], and the electromagnetic corrections are under investigation [19]. A particular combination of form factors of the vector currents has been studied at two-loop order in [20].

2.3 Future experiments

The semileptonic K_{l3} decays will be measured in the near future at DAFNE [21]. Of course, it will be very interesting to compare the data with the prediction (8).

3 Elastic $\pi\pi$ -scattering

The interplay between theoretical and experimental aspects of elastic $\pi\pi$ scattering is illustrated in figure 1. On the theoretical side, Weinberg's calculation [22] of the scattering amplitude at leading order in the low-energy expansion gives for the isospin zero S-wave scattering length the value $a_{l=0}^{I=0} = 0.16$ in units of the charged pion mass. This differs from the experimentally determined value [23] $a_0^0 = 0.26 \pm 0.05$ by two standard deviations. The one-loop calculation [24] enhances the leading order term to $a_0^0 = 0.20 \pm 0.01$ - the correction goes in the right direction, but the result is still on the low side as far as the present experimental value is concerned. To decide about agreement/disagreement between theory and experiment, one should i) evaluate

the scattering lengths in the theoretical framework at order p^6 , and ii) determine them more precisely experimentally. Let me first comment on the theoretical work.

Theory	Experiment
$A = \frac{s-M_\pi^2}{F_\pi^2} + O(p^4)$ \downarrow $a_0^0 = 0.16$	$K \rightarrow \pi\pi e\nu$ (30 000 decays) \downarrow $a_0^0 = 0.26 \pm 0.05$
$+ O(p^4) \downarrow$	$\text{DIRAC} \Downarrow \text{E865; KLOE}$
$a_0^0 = 0.20 \pm 0.01$	<div style="border: 1px solid black; padding: 2px; display: inline-block;">$a_0^0 = ?$</div>
$+ O(p^6) \Downarrow$	
<div style="border: 1px solid black; padding: 2px; display: inline-block;">$a_0^0 = ?$</div>	

Figure 1: Progress in the determination of the elastic $\pi\pi$ scattering amplitude. References are provided in the text.

3.1 Theoretical aspects

I consider QCD in the isospin symmetry limit $m_u = m_d \neq 0$. Elastic $\pi\pi$ scattering is then described by a single Lorentz invariant amplitude $A(s, t, u)$, that depends on the standard Mandelstam variables s, t, u . The effective lagrangian that describes this process is given by a string of terms, $\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \hbar\mathcal{L}_4 + \hbar^2\mathcal{L}_6 + \dots$, where \mathcal{L}_n contains m_1 derivatives of the pion fields and m_2 quark mass matrices, with $m_1 + 2m_2 = n$ (here, I consider the standard counting rules [1, 2]). The low-energy expansion corresponds to an expansion of the scattering amplitude in powers of \hbar ,

$$A(s, t, u) = \left\{ \begin{array}{cccc} A_2 & + & A_4 & + & A_6 & + & O(p^8) \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{tree} & & 1 \text{ loop} & & 2 \text{ loops} & & \end{array} \right. , \quad (9)$$

where A_n is of order p^n . The tree-level result [22] reads

$$A_2 = \frac{s - M_\pi^2}{F_\pi^2}, \quad (10)$$

and the one-loop expression A_4 may be found in [24]. The two-loop contribution A_6 was worked out in [25]. (A dispersive evaluation of A_6 has been performed in Ref. [26] in the framework of generalized chiral perturbation theory, see below. That calculation is not sufficient for the present purpose - what is needed for the analysis outlined below is the complete two-loop expression of A_6 as presented in [25].)

The amplitude $A_2 + A_4 + A_6$ contains several of the low-energy constants that occur in \mathcal{L}_{eff} . In \mathcal{L}_2 , there are two of them, the pion decay constant F in the chiral limit, and the parameter B , which are related to the condensate by $F^2 B = -\langle 0 | \bar{u}u | 0 \rangle$. In the loop expansion, these two parameters can be expressed in terms of the physical pion decay constant $F_\pi \simeq 92.4$ MeV and of the pion mass, $M_\pi = 139.57$ MeV. The $\pi\pi$ scattering amplitude contains, in the two-loop approximation, in addition several LEC's occurring in \mathcal{L}_4 and in \mathcal{L}_6 ,

$$\left. \begin{array}{l} \mathcal{L}_2 : F_\pi, M_\pi \\ \mathcal{L}_4 : \bar{l}_1, \bar{l}_2, \bar{l}_3, \bar{l}_4 \\ \mathcal{L}_6 : \bar{r}_1, \dots, \bar{r}_6 \end{array} \right\} \text{ occur in } \pi\pi \rightarrow \pi\pi \text{ (two-loop approximation)}. \quad (11)$$

These LEC's are not determined by chiral symmetry - they are, however, in principle calculable in QCD [27].

Once the amplitude is available in algebraic form, it is a trivial matter to evaluate the threshold parameters. To quote an example, the isospin zero S-wave scattering length is of the form

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} \left\{ 1 + c_4 x + c_6 x^2 + O(p^8) \right\}; \quad x = \frac{M_\pi^2}{16\pi^2 F_\pi^2}. \quad (12)$$

The coefficients c_4, c_6 contain the low-energy constants listed in (11). Similar formulae hold for all other threshold parameters - the explicit expressions for the scattering lengths and effective ranges of the S- and P-waves as well as for the D-wave scattering lengths at order p^6 may be found in [25]. It is clear that, before a numerical value for these parameters can be given, one needs an estimate of the low-energy constants. The calculation is under way - it is, however, quite involved: One has to solve numerically the Roy-equations

[28] with input from the high-energy absorptive part. Second, one assumes that the couplings that describe the mass dependence of the amplitude may be estimated from resonance exchange. Requiring that the experimental amplitude agrees near threshold with the chiral representation allows one finally to pin down the remaining couplings, as well as the scattering lengths a_0^0 and a_0^2 . The remaining threshold parameters may then be obtained from the Wanders sum rules [29]. The first part of the program is completed, and the report will appear soon [30]. The second part, that will allow us to predict the values of all threshold parameters, is under investigation [31].

3.2 Threshold parameters from experimental data

On the *experimental* side, several attempts are under way to improve our knowledge of the threshold parameters. The most promising ones among them are i) semileptonic K_{l4} decays with improved statistics, E865 [32] and KLOE [33], and ii) the measurement of the pionium lifetime - DIRAC [34] - that will allow one to directly determine the combination $|a_0^0 - a_0^2|$ of S-wave scattering lengths. It was one of the aims of last years workshop in Dubna [35] to discuss the precise relation between the lifetime of the pionium atom and the $\pi\pi$ scattering lengths - I refer the interested reader to the numerous contributions to that workshop for details. Let me note that recently, using the effective lagrangian framework proposed by Caswell and Lepage some time ago [36], the width of pionium in its ground state has been determined [37] at leading and next-to-leading order in isospin breaking and to all orders in the chiral expansion. This result will allow one to evaluate the combination $|a_0^0 - a_0^2|$ with high precision, provided that DIRAC determines the lifetime at the 10% level, as is foreseen [34].

3.3 Why do we wish to know the scattering lengths?

Why are we interested in a precise value of the scattering length a_0^0 ? First, it is one of the few occasions that a quantity in QCD can be predicted rather precisely - which is, of course, by itself worth checking. Second, as has been pointed out in [38], this prediction assumes that the condensate has the standard size in the chiral limit - in particular, it is assumed to be non vanishing. For this reason, the authors of Ref. [38] have reversed the argument and have set up a framework where the condensate is allowed to be small or even vanishing in the chiral limit - the so called generalized

chiral perturbation theory ⁴. Whereas the S-wave scattering lengths cannot be predicted in that framework, one may relate their size to the value of the condensate. Hence, measuring a_0^0 , a_0^2 or a combination thereof [34] may allow one to determine the nature of chiral symmetry breaking by experiment [38, 39].

4 Conclusion

Chiral perturbation theory has a wide field of applications. Many of its predictions have already been tested [6, 43, 44], and many more will be investigated in the near future, e.g. by E865 [32] in Brookhaven, by DIRAC [34] at CERN, and by DAFNE in Frascati [6].

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⁴Let me note that there is no sign for a small condensate in present lattice calculations [40]. Further interesting investigations of the small condensate scenario have been performed in Refs. [41, 42].

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